Material Model 10: Elastic-Plastic-Hydrodynamic

For completeness we give the entire derivation of this constitutive model based on radial return plasticity.

The pressure, p, deviatoric strain rate, $\dot{\varepsilon}'_{ij}$, deviatoric stress rate, $\dot{s}_{ij}$, volumetric strain rate, and $\dot{\varepsilon}_v$, are defined in Equation (19.10.1):

$$p = -\frac{1}{3}\sigma_{ij}\,\delta_{ij} \qquad \dot{\varepsilon}'_{ij} = \dot{\varepsilon}_{ij} - \frac{1}{3}\dot{\varepsilon}_v$$

$$s_{ij} = \sigma_{ij} + p\delta_{ij} \qquad \dot{\varepsilon}_v = \dot{\varepsilon}_{ij}\,\delta_{ij} \tag{19.10.1}$$

$$s_{ij}^{\nabla} = 2\mu\,\dot{\varepsilon}'_{ij} = 2G\dot{\varepsilon}'_{ij}$$

The Jaumann rate of the deviatoric stress, s_{ij}^{∇}, is given by:

$$s_{ij}^{\nabla} = \dot{s}_{ij} - s_{ip}\Omega_{pj} - s_{jp}\Omega_{pi} \tag{19.10.2}$$

First we update s_{ij}^{n} to s_{ij}^{n+1} elastically

$$^{*}s_{ij}^{n+1} = s_{ij}^{n} + s_{ip}\Omega_{pj} + s_{jp}\Omega_{pi} + 2G\dot{\varepsilon}'_{ij}\,dt = \underbrace{s_{ij}^{n} + R_{ij}}_{s_{ij}^{R^n}} + \underbrace{2G\dot{\varepsilon}'_{ij}\,dt}_{2G\Delta\varepsilon'_{ij}} \tag{19.10.3}$$

where the left superscript, *, denotes a trial stress value. The effective trial stress is defined by

$$s^{*} = \left(\frac{3}{2}\,{}^{*}s_{ij}^{n+1}\;{}^{*}s_{ij}^{n+1}\right)^{1/2} \tag{19.10.4}$$

and if s^{*} exceeds yield stress σ_y, the Von Mises flow rule:

$$\phi = \frac{1}{2}s_{ij}\,s_{ij} - \frac{\sigma_y^{\,2}}{3} \le 0 \tag{19.10.5}$$

is violated and we scale the trial stresses back to the yield surface, i.e., a radial return

$$s_{ij}^{n+1} = \frac{\sigma_y}{s^{*}}\;{}^{*}s_{ij}^{n+1} = m\,{}^{*}s_{ij}^{n+1} \tag{19.10.6}$$

The plastic strain increment can be found by subtracting the deviatoric part of the strain increment that is elastic, $\frac{1}{2G}\left(s_{ij}^{n+1} - s_{ij}^{R^n}\right)$, from the total deviatoric increment, $\Delta\varepsilon'_{ij}$, i.e.,

$$\Delta\varepsilon_{ij}^{p} = \Delta\varepsilon'_{ij} - \frac{1}{2G}\left(s_{ij}^{n+1} - s_{ij}^{R^n}\right) \tag{19.10.7}$$

Recalling that,

$$\Delta\varepsilon'_{ij} = \frac{{}^{*}s_{ij}^{n+1} - s_{ij}^{R^n}}{2G} \tag{19.10.8}$$

and substituting Equation (19.10.8) into (19.10.7) we obtain,

$$\Delta\varepsilon_{ij}^{p} = \frac{\left({}^{*}s_{ij}^{n+1} - s_{ij}^{n+1}\right)}{2G} \tag{19.10.9}$$

Substituting Equation (19.10.6)

$$s_{ij}^{n+1} = m\,{}^{*}s_{ij}^{n+1}$$

into Equation (19.10.9) gives,

$$\Delta\varepsilon_{ij}^{p} = \left(\frac{1-m}{2G}\right){}^{*}s_{ij}^{n+1} = \frac{1-m}{2Gm}s_{ij}^{n+1} = d\lambda s_{ij}^{n+1} \tag{19.10.10}$$

By definition an increment in effective plastic strain is

$$\Delta\varepsilon^{p} = \left(\frac{2}{3}\Delta\varepsilon_{ij}^{p}\ \Delta\varepsilon_{ij}^{p}\right)^{1/2} \tag{19.10.11}$$

Squaring both sides of Equation (19.10.10) leads to:

$$\Delta\varepsilon_{ij}^{p}\ \Delta\varepsilon_{ij}^{p} = \left(\frac{1-m}{2G}\right)^{2}\ {}^{*}s_{ij}^{n+1}\ \ {}^{*}s_{ij}^{n+1} \tag{19.10.12}$$

or from Equations (19.10.4) and (19.10.11):

$$\frac{3}{2}\Delta\varepsilon^{p^2} = \left(\frac{1-m}{2G}\right)^{2}\frac{2}{3}s^{*2} \tag{19.10.13}$$

Hence,

$$\therefore \Delta\varepsilon^p = \frac{1-m}{3G}s^* = \frac{s^* - \sigma_y}{3G} \tag{19.10.14}$$

where we have substituted for m from Equation (19.10.6)

$$m = \frac{\sigma_y}{s^*}$$

If isotropic hardening is assumed then:

$$\sigma_y^{n+1} = \sigma_y^n + E^p \Delta\varepsilon^p \tag{19.10.15}$$

and from Equation (19.10.14)

$$\Delta\varepsilon^p = \frac{\left(s^* - \sigma_y^{n+1}\right)}{3G} = \frac{\left(s^* - \sigma_y^n - E^p \Delta\varepsilon^p\right)}{3G} \tag{19.10.16}$$

Thus,

$$\left(3G + E^p\right)\Delta\varepsilon^p = \left(s^* - \sigma_y^n\right)$$

and solving for the incremental plastic strain gives

$$\Delta\varepsilon^p = \frac{\left(s^* - \sigma_y^n\right)}{\left(3G + E^p\right)} \tag{19.10.17}$$

The algorithm for plastic loading can now be outlined in five simple stress. If the effective trial stress exceeds the yield stress then

1. Solve for the plastic strain increment:

$$\Delta\varepsilon^p = \frac{\left(s^* - \sigma_y^n\right)}{\left(3G + E^p\right)}$$

2. Update the plastic strain:

$$\varepsilon^{p^{n+1}} = \varepsilon^{p^n} + \Delta\varepsilon^p\,.$$

3. Update the yield stress:

$$\sigma_y^{n+1} = \sigma_y^n + E^p \Delta\varepsilon^p$$

4. Compute the scale factor using the yield strength at time $n+1$:

$$m = \frac{\sigma_y^{n+1}}{s^*}$$

5. Radial return the deviatoric stresses to the yield surface:

$$s_{ij}^{n+1} = m^* s_{ij}^{n+1}$$

Material Model 11: Elastic-Plastic With Thermal Softening

Steinberg and Guinan [1978] developed this model for treating plasticity at high strain rates (10^5 s^{-1}) where enhancement of the yield strength due to strain rate effects is saturated out.

Both the shear modulus G and yield strength σ_y increase with pressure but decrease with temperature. As a melt temperature is reached, these quantities approach zero. We define the shear modulus before the material melts as

$$G = G_0 \left[1 + bpV^{1/3} - h \left(\frac{E - E_c}{3R'} - 300 \right) \right] e^{-\frac{fE}{E_m - E}} \tag{19.11.1}$$

where G_0, b, h, and f are input parameters, E_c is the cold compression energy:

$$E_c(X) = \int_0^x pdx - \frac{900\, R' \exp(ax)}{(1-X)^{2\left(\gamma_o - a - \frac{1}{2}\right)}}, \tag{19.11.2}$$

where

$$X = 1 - V, \tag{19.11.3}$$

and E_m is the melting energy:

$$E_m(X) = E_c(X) + 3R' T_m(X) \tag{19.11.4}$$

which is a function of the melting temperature $T_m(X)$:

$$T_m(X) = \frac{T_{mo} \exp(2aX)}{(1-X)^{2\left(\gamma_o - a - 1/3\right)}} \tag{19.11.5}$$

and the melting temperature T_{mo} at $\rho = \rho_0$. The constants γ_0 and a are input parameters. In the above equation, R' is defined by