Material Model 76: General Viscoelastic

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}\left(t-\tau\right)\frac{\partial \varepsilon_{kl}}{\partial \tau}d\tau \tag{19.76.1}$$

where $g_{ijkl}(t-\tau)$ is the relaxation function.

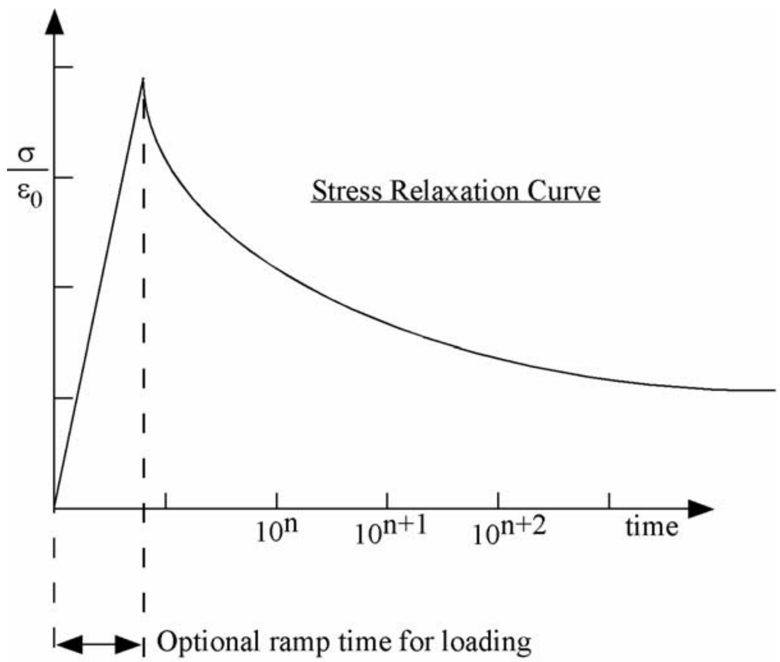

Figure 19.76.1. Relaxation curve. This curve defines stress versus time where time is defined on a logarithmic scale. For best results, the points defined in the load curve should be equally spaced on the logarithmic scale. Furthermore, the load curve should be smooth and defined in the positive quadrant. If nonphysical values are determined by least squares fit, LS-DYNA will terminate with an error message after the initialization phase is completed. If the ramp time for loading is included, then the relaxation which occurs during the loading phase is taken into account. This effect may or may not be important.

If we wish to include only simple rate effects for the deviatoric stresses, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \sum_{m=1}^{N} G_m e^{-\beta_m t} \tag{19.76.2}$$

We characterize this in the input by shear modulii, G_i, and decay constants, β_i. An arbitrary number of terms, up to 6, may be used when applying the viscoelastic model.

For volumetric relaxation, the relaxation function is also represented by the Prony series in terms of bulk modulii:

$$k(t) = \sum_{m=1}^{N} K_m e^{-\beta_{k_m} t} \qquad (19.76.3)$$

Material Model 77: Hyperviscoelastic Rubber

Material type 77 in LS-DYNA consists of two hyperelastic rubber models, a general hyperelastic rubber model and an Ogden rubber model, that can be combined optionally with a viscoelastic stress contribution. As for the rate independent part, the constitutive law is determined by a strain energy function which in this case advantageously can be expressed in terms of the principal stretches, i.e., $W = W(\lambda_1, \lambda_2, \lambda_3)$. To obtain the Cauchy stress σ_{ij}, as well as the constitutive tensor of interest, D_{ijkl}^{TC}, they are first calculated in the principal basis after which they are transformed back to the "base frame", or standard basis. The complete set of formulas is given by Crisfield [1997] and is for the sake of completeness recapitulated here.

The principal Kirchoff stress components are given by

$$\tau_{ii}^{E} = \lambda_i \frac{\partial W}{\partial \lambda_i} \text{ (no sum)}$$

that are transformed to the standard basis using the standard formula

$$\tau_{ij} = q_{ik} q_{jl} \tau_{kl}^{E} .$$

The q_{ij} are the components of the orthogonal tensor containing the eigenvectors of the principal basis. The Cauchy stress is then given by

$$\sigma_{ij} = J^{-1} \tau_{ij} ,$$

where $J = \lambda_1 \lambda_2 \lambda_3$ is the relative volume change.

The constitutive tensor that relates the rate of deformation to the Truesdell (convected) rate of Kirchoff stress can in the principal basis be expressed as

$$D_{iijj}^{TKE} = \lambda_j \frac{\partial \tau_{ii}^{E}}{\partial \lambda_j} - 2\tau_{ii}^{E} \delta_{ij}$$

$$D_{ijij}^{TKE} = \frac{\lambda_j^2 \tau_{ii}^{E} - \lambda_i^2 \tau_{jj}^{E}}{\lambda_i^2 - \lambda_j^2}, \quad i \neq j, \lambda_i \neq \lambda_j \qquad \text{(no sum)}$$

$$D_{ijij}^{TKE} = \frac{\lambda_i}{2} \left(\frac{\partial \tau_{ii}^{E}}{\partial \lambda_i} - \frac{\partial \tau_{ii}^{E}}{\partial \lambda_j} \right), \quad i \neq j, \lambda_i = \lambda_j$$