Material Model 133: Barlat_YLD2000

The yield condition for this material can be written

$$
\begin{aligned}
f(\boldsymbol{\sigma},\boldsymbol{\alpha},\varepsilon_p) = \sigma_{\text{eff}}(\sigma_{xx}-2\alpha_{xx}-\alpha_{yy},\sigma_{yy}-2\alpha_{yy}-\alpha_{xx},\sigma_{xy}-\alpha_{xy}) - \\
\sigma_Y^t(\varepsilon_p,\dot{\varepsilon}_p,\beta) \le 0
\end{aligned}
\tag{19.133.1}
$$

where

$$
\begin{aligned}
\sigma_{\text{eff}}(s_{xx},s_{yy},s_{xy}) &= \left(\frac{1}{2}(\phi'+\phi'')\right)^{1/a} \\
\phi' &= \left|X'_1 - X'_2\right|^a \\
\phi'' &= \left|2X''_1 + X''_2\right|^a + \left|X''_1 + 2X''_2\right|^a
\end{aligned}
\tag{19.133.2}
$$

The X'_i and X''_i are the eigenvalues of X'_{ij} and X''_{ij} and are given by

$$
\begin{aligned}
X'_1 &= \frac{1}{2}\left(X'_{11}+X'_{22}+\sqrt{(X'_{11}-X'_{22})^2+4X'^2_{12}}\right) \\
X'_2 &= \frac{1}{2}\left(X'_{11}+X'_{22}-\sqrt{(X'_{11}-X'_{22})^2+4X'^2_{12}}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
X''_1 &= \frac{1}{2}\left(X''_{11}+X''_{22}+\sqrt{(X''_{11}-X''_{22})^2+4X''^2_{12}}\right) \\
X''_2 &= \frac{1}{2}\left(X''_{11}+X''_{22}-\sqrt{(X''_{11}-X''_{22})^2+4X''^2_{12}}\right)
\end{aligned}
$$

respectively. The X'_{ij} and X''_{ij} are given by

$$
\begin{pmatrix} X'_{11} \\ X'_{22} \\ X'_{12} \end{pmatrix} = \begin{pmatrix} L'_{11} & L'_{12} & 0 \\ L'_{21} & L'_{22} & 0 \\ 0 & 0 & L'_{33} \end{pmatrix} \begin{pmatrix} s_{xx} \\ s_{yy} \\ s_{xy} \end{pmatrix} \qquad \begin{pmatrix} X''_{11} \\ X''_{22} \\ X''_{12} \end{pmatrix} = \begin{pmatrix} L''_{11} & L''_{12} & 0 \\ L''_{21} & L''_{22} & 0 \\ 0 & 0 & L''_{33} \end{pmatrix} \begin{pmatrix} s_{xx} \\ s_{yy} \\ s_{xy} \end{pmatrix}
$$

where

$$
\begin{pmatrix} L'_{11} \\ L'_{12} \\ L'_{21} \\ L'_{22} \\ L'_{33} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_7 \end{pmatrix} \qquad \begin{pmatrix} L''_{11} \\ L''_{12} \\ L''_{21} \\ L''_{22} \\ L''_{33} \end{pmatrix} = \frac{1}{9} \begin{pmatrix} -2 & 2 & 8 & -2 & 0 \\ 1 & -4 & -4 & 4 & 0 \\ 4 & -4 & -4 & 1 & 0 \\ -2 & 8 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{pmatrix} \begin{pmatrix} \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_8 \end{pmatrix}
$$

where α_1 to α_8 are the parameters that determine the shape of the yield surface.

The yield stress is expressed as

$$\sigma_Y^t(\varepsilon_p, \dot{\varepsilon}_p, \beta) = \sigma_Y^v(\varepsilon_p, \dot{\varepsilon}_p) + \beta(\sigma_0 - \sigma_Y^v(\varepsilon_p, \dot{\varepsilon}_p)) \qquad (19.133.3)$$

where β determines the fraction kinematic hardening and σ_0 is the initial yield stress. The yield stress for purely isotropic hardening is given by

$$\sigma_Y^v(\varepsilon_p, \dot{\varepsilon}_p) = \sigma_Y(\varepsilon_p)\left(1 + \left\{\frac{\dot{\varepsilon}_p}{C}\right\}^{1/p}\right) \qquad (19.133.4)$$

where C and p are the Cowper-Symonds material parameters for strain rate effects.

The evolution of back stress is given by

$$\dot{\boldsymbol{\alpha}} = \dot{\lambda}\beta \frac{\left(\dfrac{\partial \sigma_Y}{\partial \varepsilon_p}\left(1 + \left\{\dfrac{\dot{\varepsilon}_p}{C}\right\}^{1/p}\right) + \sigma_Y \dfrac{1}{pC\Delta t}\left\{\dfrac{\dot{\varepsilon}_p}{C}\right\}^{1/p-1}\right)}{\dfrac{\partial \sigma_{\text{eff}}}{\partial \mathbf{s}} \cdot \dfrac{\partial \sigma_{\text{eff}}}{\partial \mathbf{s}}} \frac{\partial \sigma_{\text{eff}}}{\partial \mathbf{s}} \qquad (19.133.5)$$

where Δt is the current time step size and $\dot{\lambda}$ is the rate of plastic strain multiplier.

For the plastic return algorithms, the gradient of effective stress is computed as

$$\begin{pmatrix} \dfrac{\partial \sigma_{\text{eff}}}{\partial s_{xx}} \\ \dfrac{\partial \sigma_{\text{eff}}}{\partial s_{yy}} \\ \dfrac{\partial \sigma_{\text{eff}}}{\partial s_{xy}} \end{pmatrix} = \frac{a\sigma_{\text{eff}}^{a-1}}{2} \begin{pmatrix} L'_{11} & L'_{21} & 0 \\ L'_{12} & L'_{22} & 0 \\ 0 & 0 & L'_{33} \end{pmatrix} \begin{pmatrix} \dfrac{\partial \phi'}{\partial X'_{11}} \\ \dfrac{\partial \phi'}{\partial X'_{22}} \\ \dfrac{\partial \phi'}{\partial X'_{12}} \end{pmatrix} + \frac{a\sigma_{\text{eff}}^{a-1}}{2} \begin{pmatrix} L''_{11} & L''_{21} & 0 \\ L''_{12} & L''_{22} & 0 \\ 0 & 0 & L''_{33} \end{pmatrix} \begin{pmatrix} \dfrac{\partial \phi''}{\partial X''_{11}} \\ \dfrac{\partial \phi''}{\partial X''_{22}} \\ \dfrac{\partial \phi''}{\partial X''_{12}} \end{pmatrix}$$

with the aid of

$$\frac{\partial \phi'}{\partial X'_{ij}} = a(X'_1 - X'_2)^{a-1} \frac{\partial(X'_1 - X'_2)}{\partial X'_{ij}}$$

$$\begin{aligned} \frac{\partial \phi''}{\partial X''_{ij}} = {} & a\left|2X''_1 + X''_2\right|^{a-1} \operatorname{sgn}(2X''_1 + X''_2) \frac{\partial(2X''_1 + X''_2)}{\partial X''_{ij}} + \\ & a\left|2X''_2 + X''_1\right|^{a-1} \operatorname{sgn}(2X''_2 + X''_1) \frac{\partial(2X''_2 + X''_1)}{\partial X''_{ij}} \end{aligned}$$

and

$$\frac{\partial\left(X'_1 - X'_2\right)}{\partial X'_{11}} = \frac{X'_{11} - X'_{22}}{\sqrt{(X'_{11} - X'_{22})^2 + 4X'^2_{12}}}$$

$$\frac{\partial\left(X'_1 - X'_2\right)}{\partial X'_{22}} = \frac{X'_{22} - X'_{11}}{\sqrt{(X'_{11} - X'_{22})^2 + 4X'^2_{12}}}$$

$$\frac{\partial\left(X'_1 - X'_2\right)}{\partial X'_{12}} = \frac{4X'_{12}}{\sqrt{(X'_{11} - X'_{22})^2 + 4X'^2_{12}}}$$

$$\frac{\partial\left(2X''_1 + X''_2\right)}{\partial X''_{11}} = \frac{3}{2} + \frac{1}{2}\frac{X''_{11} - X''_{22}}{\sqrt{(X''_{11} - X''_{22})^2 + 4X''^2_{12}}}$$

$$\frac{\partial\left(2X''_1 + X''_2\right)}{\partial X''_{22}} = \frac{3}{2} + \frac{1}{2}\frac{X''_{22} - X''_{11}}{\sqrt{(X''_{11} - X''_{22})^2 + 4X''^2_{12}}}$$

$$\frac{\partial\left(2X''_1 + X''_2\right)}{\partial X''_{12}} = \frac{2X''_{12}}{\sqrt{(X''_{11} - X''_{22})^2 + 4X''^2_{12}}}$$

$$\frac{\partial\left(2X''_2 + X''_1\right)}{\partial X''_{11}} = \frac{3}{2} - \frac{1}{2}\frac{X''_{11} - X''_{22}}{\sqrt{(X''_{11} - X''_{22})^2 + 4X''^2_{12}}}$$

$$\frac{\partial\left(2X''_2 + X''_1\right)}{\partial X''_{22}} = \frac{3}{2} - \frac{1}{2}\frac{X''_{22} - X''_{11}}{\sqrt{(X''_{11} - X''_{22})^2 + 4X''^2_{12}}}$$

$$\frac{\partial\left(2X''_2 + X''_1\right)}{\partial X''_{12}} = -\frac{2X''_{12}}{\sqrt{(X''_{11} - X''_{22})^2 + 4X''^2_{12}}}$$

The algorithm for the plane stress update as well as the formula for the tangent modulus is given in detail in Section 19.36.1 and is not repeated here.

19.133.1 Closest point projection algorithm

This section describes shortly the closest point projection algorithm that was implemented to improve accuracy, hence the implicit performance, of the model. The closest point projection comes down to solving the following system of equations

$$\mathbf{f}_1 = \mathbf{t} + \mathbf{A}\boldsymbol{\alpha} + (\sigma_Y^t(\Delta\lambda) - \sigma_Y^t(0))\mathbf{h} - \boldsymbol{\sigma}^{\text{trial}}(\Delta\varepsilon_{33}) + 2G\Delta\lambda\mathbf{D}\nabla\sigma_{\text{eff}}(\mathbf{t}) = \mathbf{0}$$

$$f_2 = -\sigma_{\text{eff}}(\mathbf{t}) + \sigma_Y^t(\Delta\lambda) = 0$$

$$f_3 = \sigma_{33}^{\text{trial}}(\Delta\varepsilon_{33}) + 2G\Delta\lambda(\nabla\sigma_{\text{eff}}^1(\mathbf{t}) + \nabla\sigma_{\text{eff}}^2(\mathbf{t})) = 0$$

where

$$\mathbf{h} = \frac{\beta}{\nabla\sigma_{\text{eff}}(\mathbf{t})^T\mathbf{B}\nabla\sigma_{\text{eff}}(\mathbf{t})}\mathbf{B}\nabla\sigma_{\text{eff}}(\mathbf{t})$$

in terms of the unknown variables $\mathbf{t}$ (stress), $\Delta\varepsilon_{33}$ (thickness strain increment) and $\Delta\lambda$ (plastic strain increment). In the above

$$\mathbf{D} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0.5 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 2 & 1 & \\ 1 & 2 & \\ & & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & 1 & \\ 1 & 2 & \\ & & 0.5 \end{bmatrix} \quad H = \frac{\partial\sigma_Y^t}{\partial\varepsilon_p}.$$

This system of equations is solved using a Newton method with an additional line search for robustness. Using the notation

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}_1 \\ f_2 \\ f_3 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} \mathbf{t} \\ \Delta\lambda \\ \Delta\varepsilon_{33} \end{bmatrix},$$

a Newton step is completed as

$$\mathbf{x}^+ = \mathbf{x}^- - s\left(\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right)^{-1}\mathbf{f}$$

for a step size $s \le 1$ chosen such that the norm of the objective function is decreasing. The gradient of the objective function is given by

$$\nabla\mathbf{f} = \nabla\mathbf{f}_{1-\beta} + \nabla\mathbf{f}_\beta$$

where

$$\nabla\mathbf{f}_{1-\beta} = \begin{bmatrix} \mathbf{I} + 2G\Delta\lambda\mathbf{D}\nabla^2\sigma_{\text{eff}} & 2G\mathbf{D}\nabla\sigma_{\text{eff}} & -\mathbf{C}_3 \\ -(\nabla\sigma_{\text{eff}})^T & H & 0 \\ 2G\Delta\lambda\mathbf{e}^T\nabla^2\sigma_{\text{eff}} & 2G\mathbf{e}^T\nabla\sigma_{\text{eff}} & C_{33} \end{bmatrix}$$

$$\nabla\mathbf{f}_\beta = \begin{bmatrix} \Delta\lambda H\dfrac{\partial\mathbf{h}}{\partial\mathbf{t}} & H\mathbf{h} & 0 \\ \mathbf{0}^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$\mathbf{e} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \mathbf{C}_3 = (K - 2G/3)\mathbf{e} \quad C_{33} = (K + 4G/3)\,.$$

G and K stands for the shear and bulk modulus, respectively. This algorithm requires computation of the effective stress hessian. The derivation of this is quite straightforward but the expression for it is rather long and is hence omitted in this report.

Material Model 134: Viscoelastic Fabric

The viscoelastic fabric model is a variation on the general viscoelastic Material Model 76. This model is valid for 3 and 4 node membrane elements only and is strongly recommended for modeling isotropic viscoelastic fabrics where wrinkling may be a problem. For thin fabrics, buckling can result in an inability to support compressive stresses; thus, a flag is included for this option. If bending stresses are important, use a shell formulation with Model 76.

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}\left(t-\tau\right)\frac{\partial \varepsilon_{kl}}{\partial \tau}d\tau \tag{19.134.1}$$

where $g_{ijkl}(t-\tau)$ is the relaxation function.

If we wish to include only simple rate effects for the deviatoric stresses, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \sum_{m=1}^{N} G_m e^{-\beta_m t} \tag{19.134.2}$$

We characterize this in the input by shear modulii, G_i, and decay constants, β_i. An arbitrary number of terms, up to 6, may be used when applying the viscoelastic model.

For volumetric relaxation, the relaxation function is also represented by the Prony series in terms of bulk modulii:

$$k(t) = \sum_{m=1}^{N} K_m e^{-\beta_{k_m} t} \tag{19.134.3}$$

Material Model 139: Modified Force Limited

This material model is available for the Belytschko resultant beam element only. Plastic hinges form at the ends of the beam when the moment reaches the plastic moment. The plastic moment versus rotation relationship is specified by the user in the form of a load curve and scale factor. The points of the load curve are (plastic rotation in radians, plastic moment). Both quantities should be positive for all points, with the first point being (zero, initial plastic moment). Within this constraint any form of characteristic may be used, including flat or falling curves. Different load curves and scale factors may be specified at each node and about each of the local s and t axes.